Research on the Inverse Problem of Electrical Impedance Tomography Based on Improved Regularization

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***Abstract*—Regularization is an important method to improve the stability and image resolution of electrical impedance tomography. This paper proposes an improved regularization method based on the traditional Tikhonov and diagonal weight regularization method（DWRM）. Firstly, the ill-posedness of inverse problem is analyzed in view of sensitivity, then, the singular value decomposition (SVD) is discussed to analyze the regularization method. Finally, some simulation examples are further analyzed to verify the validation of the proposed method. Results indicate that the method obtains higher image resolution and anti-noise ability than the traditional regularization.**

***Index Terms*—electrical impedance tomography, inverse problem, regularization, resolution, anti-noise ability**

I. INTRODUCTION

Electrical impedance tomography(EIT) is a new biological structural and functional imaging technology, which is nondestructive and inexpensive [1]. Many researches have been done on EIT and it also has been used in other industrial field [2]. As EIT image reconstruction is a nonlinear inverse problem with serious ill-posedness, the regularization is needed to be imposed on the inverse problem to obtain stable solution. Regularization works as a penalty term on the objective function, which usually contains the prior information of the objective area. Different penalty terms would derive different regularization methods, including Tikhonov regularization [3], total variation regularization [4] and so on. Many efforts have been done to deal with the ill-posedness of inverse problem, while the imaging resolution and the anti-noise ability of EIT are still needed to be improved further. Therefore, in the paper, an improved regularization method is proposed, which combine the advantage of the Tikhonov regularization method with diagonal weight regularization method. Meanwhile, the SVD analysis and some simulation examples are used to verify the better resolution and the anti-noise ability of the improved regularization method.

II. THE T-D REGULARIZATION

In the complete electrode model of EIT, the electrode potential is sensitive to the change of surface resistivity, while less sensitive to the internal ones. It is inevitable that the Jacobian matrix of electrode potential on the resistivity has large condition number, which would result in ill-posed. In addition, the ill-posedness would magnify the error, and cause the solution unstable and inaccurate.

The mathematical model of Tikhonov regularization on the EIT inverse problem can be expressed as Eq.(1). It makes the solution stable with the penalty of $\alpha\left\|\boldsymbol{L}\cdot(\boldsymbol{\rho}-\boldsymbol{\rho}^{(0)})\right\|^2$.

$$\min E(\boldsymbol{\rho})=\left\|\boldsymbol{U}(\boldsymbol{\rho})-\boldsymbol{V}\right\|^2+\alpha\left\|\boldsymbol{L}\cdot(\boldsymbol{\rho}-\boldsymbol{\rho}^{(0)})\right\|^2 \tag{1}$$

And the diagonal weight regularization method can be expressed by

$$\min E(\boldsymbol{\rho})=\left\|\boldsymbol{U}(\boldsymbol{\rho})-\boldsymbol{V}\right\|^2+\beta\left\|\boldsymbol{\varLambda}\boldsymbol{\rho}\right\|^2 \tag{2}$$

This paper improves the Tikhonov regularization by combining DWRM, which is called T-D regularization.

$$\min E(\boldsymbol{\rho})=\left\|\boldsymbol{U}(\boldsymbol{\rho})-\boldsymbol{V}\right\|^2+\alpha\left\|\boldsymbol{L}\cdot(\boldsymbol{\rho}-\boldsymbol{\rho}^{(0)})\right\|^2+\beta\left\|\boldsymbol{\varLambda}\boldsymbol{\rho}\right\|^2 \tag{3}$$

Where, $\boldsymbol{\rho}^{(0)}$ is the initial solution, α and β are regularization parameters, $\boldsymbol{L}$ (identity matrix) and $\boldsymbol{\varLambda}$ are regularization matrixes and $\boldsymbol{\varLambda}^{\mathrm{T}}\boldsymbol{\varLambda}=diag(\boldsymbol{J}^{\mathrm{T}}\boldsymbol{J})$, $\boldsymbol{U}(\boldsymbol{\rho})$ is the calculation potentials by the forward problem, $\boldsymbol{V}$ is the measurements.

Newton iterative method is used to solve the inverse problem (3), which derives

$$\begin{aligned}\boldsymbol{\rho}^{(k+1)}&=\boldsymbol{\rho}^{(k)}-\left[\boldsymbol{J}_k^{\mathrm{T}}\boldsymbol{J}_k+\alpha\boldsymbol{L}^{\mathrm{T}}\boldsymbol{L}+\beta\boldsymbol{\varLambda}^{\mathrm{T}}\boldsymbol{\varLambda}\right]^{-1}\cdot\\&\quad\left[\boldsymbol{J}_k^{\mathrm{T}}(\boldsymbol{U}(\boldsymbol{\rho}^{(k)})-\boldsymbol{V})+\alpha\boldsymbol{L}^{\mathrm{T}}\boldsymbol{L}(\boldsymbol{\rho}^{(k)}-\boldsymbol{\rho}^{(0)})+\beta\boldsymbol{\varLambda}^{\mathrm{T}}\boldsymbol{\varLambda}\boldsymbol{\rho}^{(k)}\right]\end{aligned} \tag{4}$$

In each iteration, define a positive linear operator $\boldsymbol{A}\in\mathbb{R}^{u\times n}$ and do the SVD on $\boldsymbol{A}$

$$\boldsymbol{A}=\boldsymbol{P}\boldsymbol{D}\boldsymbol{Q}^{\mathrm{T}} \tag{5}$$

Where, $\boldsymbol{P}\in\mathbb{R}^{u\times u}$ and $\boldsymbol{Q}\in\mathbb{R}^{n\times n}$ are orthogonal matrixes, $\boldsymbol{D}\in\mathbb{R}^{u\times n}$ is a diagonal matrix, and the diagonal elements d_j are the diagonal singular values of $\boldsymbol{A}$.

In the iterative process, combining Eq.(3) with Eq.(5) and ignore the initial information, which derives

$$\begin{aligned}E(\boldsymbol{\rho})&=\left\|\boldsymbol{A}\boldsymbol{\rho}-\boldsymbol{V}\right\|^2+\alpha\left\|\boldsymbol{L}\boldsymbol{\rho}\right\|^2+\beta\left\|\boldsymbol{\varLambda}\boldsymbol{\rho}\right\|^2\\&=\sum_{j=1}^{r}(d_j{}^2+\alpha L_j{}^2+\beta\varLambda_j{}^2)\left[a_j-\frac{d_jV_j'}{d_j{}^2+\alpha L_j{}^2+\beta\varLambda_j{}^2}\right]^2\\&\quad-\sum_{j=1}^{r}\frac{(d_jV_j')^2}{d_j{}^2+\alpha L_j{}^2+\beta\varLambda_j{}^2}+\alpha\sum_{j=1}^{n}(L_ja_j)^2+\beta\sum_{j=1}^{n}(\varLambda_ja_j)^2\\&\quad+\sum_{j=1}^{k}(V_j')^2\end{aligned} \tag{6}$$

And the solution of the inverse problem can be obtained

$$\begin{aligned}\boldsymbol{\rho} = \boldsymbol{c}^{+}\boldsymbol{V} &= \boldsymbol{A}^{+}\boldsymbol{Q}\boldsymbol{D}^{+}\boldsymbol{P}^{\mathrm{T}}\boldsymbol{V} \\ &= \sum_{j=1}^{r}\frac{d_j^{\,2}}{d_j^{\,2}+\alpha L_j^{\,2}+\beta\Lambda_j^{\,2}}\frac{\boldsymbol{P}_j^{\mathrm{T}}\boldsymbol{V}\boldsymbol{Q}_j}{d_j}\end{aligned} \tag{7}$$

As it can be seen from (7) that the T-D regularization makes different degree of modification on the singular value, while the traditional Tikhonov regularization produces the same degree of modification. This character would make a better stability and anti-noise ability of the solution. Fig.1 gives the singular value distribution of $\boldsymbol{A}$ in Newton iterative process, it is obvious that the proposed T-D regularization method has modified singular value and obtained a small condition number than it without regularization, which are about 10^5 and 10^{16} respectively.

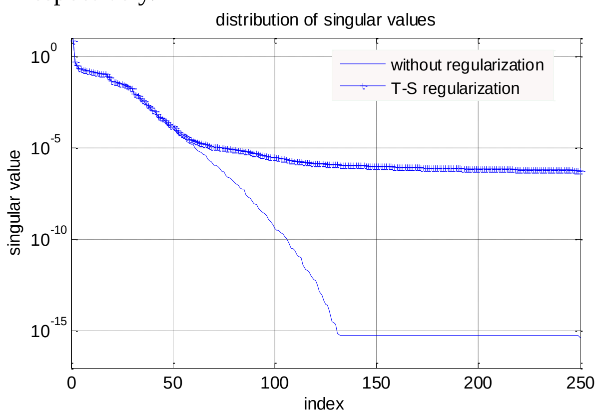


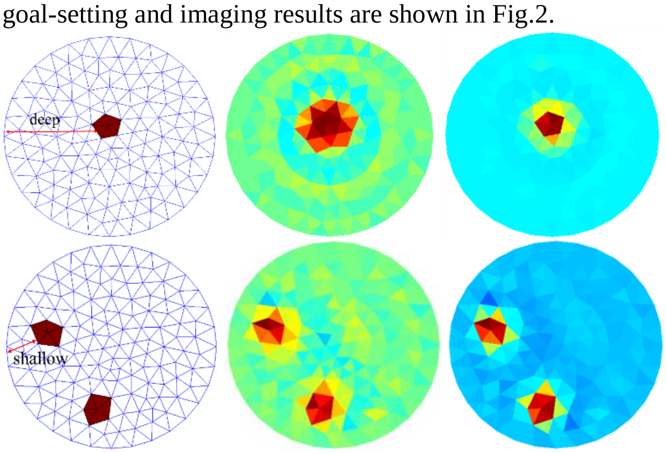
Fig.1 The singular value distribution of the operator

III. Simulation Analysis

The simulated complete electrode model of EIT is used to verify the performance of T-D regularization, which has 250 elements, 142 nodes and 16 electrodes on the boundary. The contrast of background resistivity and target resistivity is 1:5. Some comparison analysis were carried out as follows.

A. Resolution analysis

In order to compare the imaging resolution of Tikhonov and T-D regularization. We set one target in the deep of the area and the two targets in the shallow position. The diagram of goal-setting and imaging results are shown in Fig.2.



(a) The targets-setting (b) Tikhonov (c) T-D

Fig.2 The imaging results of different regularization methods

It can be seen from Fig.2 that the imaging results of T-D regularization have a clear outline between the targets and background. It also shows that a higher resolution can be obtained by T-D regularization.

B. Anti-noise ability analysis

As the inverse problem is seriously ill-posed, any slight noise would cause a great error in solution. A good regularization algorithm should not only have a good solution stability, but also a high anti-noise ability. This paper applies some random error on $\boldsymbol{V}$ to verify the anti-noise ability of different regularization, the signal-to-noise ratio (SNR) is about 30dB. Imaging results with noise of different regularization methods are shown as Fig.3.

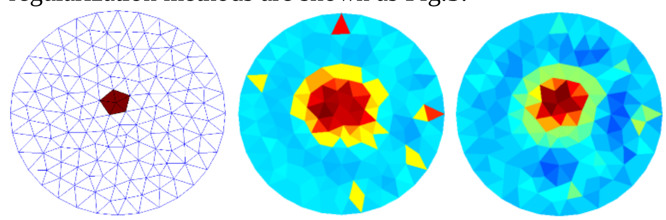

(a) The targets-setting (b) Tikhonov (c) T-D

Fig.3 The imaging results with noise of different regularization methods

It is obvious that the resolution get worse after applying noise, but it also shows a higher resolution we can obtain by T-D regularization.

IV. Conclusion

An improved regularization which combine the Tikhonov regularization and SRTM regularization is proposed in the paper. The good performance of T-D method is verified through SVD analysis and different preset target case discussions. It indicates that the T-D regularization can reduce the condition number of inverse operator. Besides, the method can not only improve the resolution but also the anti-noise ability of the reconstruction, particularly when the target is far from the boundary of the calculation area.

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